The geometry of one-weight linear codes

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Résumé

The interplay between coding theory and finite geometry has long been a field of research; see e.g. (4). At the heart of this relationship lies the correspondence between the metric properties of a code and the combinatorial properties of a corresponding point set in a projective space.
One of the main applications of this perspective can be found in the theory of one-weight codes. A one-weight code, or constant-weight code, is an error-correcting code in which all non-zero codewords share the same weight. In 1984, Bonisoli (3) provided a classification of one-weight linear codes by leveraging the connection between codes equipped with the Hamming metric and projective systems. More recently, similar geometric techniques have been applied to the study of linear codes in the rank metric; see e.g. (1).

This talk explores the evolution of this geometric interpretation, moving from classical Hamming-metric codes to matrix rank-metric codes. We present a complete characterization of nondegenerate one-weight vector rank-metric codes and illustrate why such a classification does not extend to the sum-rank metric. We also introduce a new geometric framework for matrix rank-metric codes (2) and share preliminary results on one-weight codes in this setting.

References

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